MPI-Ph/94-81
UdeM-GPP-TH-94-09
DESY 94–201
hep-ph/9411386
October 1994

A Note on the QCD Corrections to Forward–Backward Asymmetries of Heavy–Quark Jets in Z Decays

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ABSTRACT

The measurement of the forward-backward asymmetries of heavy quarks provides one of the most precise determinations of $\sin^2\theta_W$ in Z decays. We discuss in detail the one–loop QCD radiative corrections to these asymmetries. Results are given for single heavy–quark jet asymmetries and asymmetries of the thrust axis, as well as for heavy–quark two–jet final states.

1. Introduction

The heavy quark sector, especially b-quark physics, has received much attention in the high–precision analyses of the electroweak part of the Standard Model which are being performed in Z decays at LEP and SLC [for reviews see Ref. [1]]. In particular, the measurement of the forward-backward asymmetry for b-quark production A_{FB}^{b} provides one of the most precise determinations of the electroweak mixing angle $\sin^2\theta_W$ in Z decays [2]. The precision of the experimental data is so high that higher–order electroweak and QCD corrections must be incorporated properly.

The photonic and the genuine electroweak radiative corrections to quark forward–backward asymmetries A_{FB}^{Q} have been discussed thoroughly in the literature, Ref. [3]. Many features of the radiative corrections due to strong interactions have also been presented in the past [4-13]. At the one–loop level the QCD corrections consist of virtual quark–vertex corrections and bremsstrahlung corrections where one of the quarks emits an additional gluon in the final state. This bremsstrahlung correction will depend in a sensitive way on the quark–jet properties, i.e. the definition of the jet axis and the maximum invariant mass allowed for the final–state jets.

In this paper we will give a systematic and complete overview of the one–loop QCD corrections to the forward–backward asymmetries for heavy–quark production in e^+e^- annihilation near the Z resonance. We will complete the theoretical analysis and also correct some erroneous points in the literature. We will focus on the quark–mass effects; with increasing accuracy of the LEP experiments, these effects become more and more relevant for the production of bottom and charm quarks.

The results will be presented for two different definitions of the reference direction: the heavy quark axis which is usually used in analytical calculations [“single quark–jet asymmetry”], and the thrust axis, corresponding to the direction of the most energetic jet, which is more relevant from the experimental viewpoint. We will also present results for forward–backward asymmetries of two–jet final states in which the invariant mass of the jets is less than a fraction $\sqrt{y}$ of the total energy. In these two–jet asymmetries, the QCD corrections are much smaller than for the total (single quark–jet and thrust–axis) asymmetries. Two–jet asymmetries are therefore very useful to minimize the errors due to strong interactions in high–statistics measurements.

The paper is organized as follows. In the next section, we will summarize, for the sake of completeness, the tree–level results for the forward–backward asymmetries. In section 3, the QCD corrections will be discussed for the case in which the reference directions are either the quark or the thrust axes, and the results for the single quark–jet and thrust–axis asymmetries will be presented. In section 4, we will define the two–jet forward–backward asymmetries; results of the QCD corrections will be given for the experimentally useful range of invariant masses of the jets in the final state. A short summary in section 5 will conclude the paper.

2. Basic set-up

The differential cross section for the process $e^+e^- \to \bar{f}f$ is a binomial in $\cos\theta$, where θ is the angle between e^- and f,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\sigma_U + \frac{3}{4}\sin^2\theta\sigma_L + \frac{3}{4}\cos\theta\sigma_F \tag{1}$$

$U(L)$ denote the contributions of unpolarized (longitudinally polarized) γ/Z gauge bosons along the reference axis, and F denotes the difference between right and left polarizations. The total cross section is the sum of σ_U and σ_L,

$$\sigma_T = \sigma_U + \sigma_L \tag{2}$$

The forward–backward asymmetry for the full angular range can be expressed by the ratio of σ_F to σ_T,

$$A^f_{FB} = \frac{3}{4}\frac{\sigma_F}{\sigma_T} \tag{3}$$

In the Born approximation, the parton cross sections σ_T and σ_F are given by

$$\begin{aligned} \sigma_T &= \beta\tfrac{3-\beta^2}{2}\sigma_{VV} + \beta^3\sigma_{AA} \\ \sigma_F &= \beta^2\sigma_{VA} \end{aligned} \tag{4}$$

with $\beta = \sqrt{1-4m_f^2/s} = \sqrt{1-\mu^2}$ being the velocity of the final quarks. [The quark mass, denoted by m_f, will later be defined more precisely.] The phase space suppression and the dynamical $\mathcal{P}$–wave suppression are entirely taken account of by the velocity–dependent coefficients. In terms of the electric charge of the fermion e_f and the vector and axial-vector couplings of the fermion to the Z boson [$I_f^{3L} = \pm 1/2$ is the weak isospin, and $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$],

$$v_f = 2I_f^{3L} - 4s_W^2 e_f \quad \text{and} \quad a_f = 2I_f^{3L}$$

the cross sections, for quark final states, can be written as

$$\begin{aligned} \sigma_{VV} &= \frac{4\pi\alpha^2(s)}{s}e_e^2e_f^2 + \frac{G_F\alpha(s)}{\sqrt{2}}\frac{M_Z^2(s-M_Z^2)}{D_Z^2}e_ee_fv_ev_f + \frac{G_F^2}{32\pi}\frac{M_Z^4 s}{D_Z^2}(v_e^2+a_e^2)v_f^2 \\ \sigma_{AA} &= \frac{G_F^2}{32\pi}\frac{M_Z^4 s}{D_Z^2}(v_e^2+a_e^2)a_f^2 \\ \sigma_{VA} &= \frac{G_F\alpha(s)}{\sqrt{2}}\frac{M_Z^2(s-M_Z^2)}{D_Z^2}e_ee_fa_ea_f + \frac{G_F^2}{32\pi}\frac{M_Z^4 s}{D_Z^2}v_ev_fa_ea_f \end{aligned} \tag{5}$$

where, including the energy–dependent Z–boson width, D_Z^2 is given by the Breit–Wigner form $D_Z^2 = (s-M_Z^2)^2 + (s\Gamma_Z/M_Z)^2$.

On top of the Z resonance, the forward–backward asymmetry is dominated by the pure Z exchange amplitude so that in the Born approximation

$$A^f_{FB/0} = \frac{3}{4}\,\frac{2v_ea_e}{v_e^2+a_e^2}\,\frac{2\beta v_fa_f}{\frac{1}{2}(3-\beta^2)v_f^2+\beta^2a_f^2} \tag{6}$$

3. Asymmetries of single quark jets and thrust axis

At the one–loop level, the QCD corrections to the forward–backward asymmetry for quark–pair production can be divided into three parts: (a) the virtual vertex correction; (b) soft gluon bremsstrahlung [the additional gluon in the final state carries an energy less than $\lambda = 2E_g/\sqrt{s} \ll 1$]; and (c) hard gluon bremsstrahlung [the gluon carries an energy larger than E_g]. The results for these corrections in Z decays are summarized below.

Virtual corrections and soft bremsstrahlung:

The contribution of the virtual corrections to the forward–backward asymmetry, A^Q_{FB}, is given by

$$\delta A^Q_{FB}|_{\rm virtual} = A^Q_{FB/0} \times \frac{2\alpha_s}{3\pi} \frac{2\beta^2 a_Q^2 - 3v_Q^2}{\frac{3-\beta^2}{2}v_Q^2 + \beta^2 a_Q^2} \frac{1-\beta^2}{\beta} \log\frac{1-\beta}{1+\beta} \tag{7}$$

The infrared singularities which are present in both σ_T and σ_F, cancel in the ratio so that $\delta A^Q_{FB}|_{\rm virtual}$ is free of any singularities. Note that for quark masses approaching zero, the virtual corrections vanish.

The soft gluon bremsstrahlung in the process $e^+e^- \to Q\bar{Q}g$, in which the additional gluon carries an energy less than $\lambda = 2E_g/\sqrt{s} \ll 1$, contributes to σ_T and σ_F by the same amount,

$$\sigma^{\rm soft\ brems}_{F,T} = \sigma_{F,T}(1+\delta_{\rm SB}) \tag{8}$$

The analytical expression of $\delta_{\rm SB}$ can be found in Ref. [5] for instance. This correction therefore cancels out from the ratio σ_F/σ_T, and soft gluon bremsstrahlung does not contribute to A^Q_{FB} to first order in α_S,

$$\delta A^Q_{FB}|_{\rm soft\ brems} = A^Q_{FB/0} \times 0 \tag{9}$$

The cancellation of the soft bremsstrahlung correction is physically plausible since the emission of soft energy quanta cannot affect the angular direction of the leading heavy quarks, a consequence of Galilei's law of inertia.

Hard bremsstrahlung:

In a first step, when analyzing the semi–inclusive process

$$e^+e^- \to Q\bar{Q}g \to Q + X$$

we assume the quark momentum to be measured precisely with the quark direction to be chosen as the reference axis. With this choice, the contribution of the hard bremsstrahlung to the forward–backward asymmetry in Z decays including mass effects, is given by

$$\delta A^Q_{FB}|_{\rm hard\ brems.} = A^Q_{FB/0} \times \frac{\alpha_s}{\pi}\left[\frac{R_F}{\beta^2} - \frac{v_Q^2 R_V + a_Q^2 R_A}{\beta\frac{3-\beta^2}{2}v_Q^2 + \beta^3 a_Q^2}\right] \tag{10}$$

with [4, 5]

$$
\begin{aligned}
R_V &= \frac{2}{3}\int\int \frac{dxd\bar{x}}{(1-x)(1-\bar{x})}\left[x^2+\bar{x}^2-\frac{1}{4}\mu^2\left(2\lambda_+ + \lambda_- - \chi p_\perp^2 - \bar{\chi}p^2 - \chi\bar{p}^2 + 2p\bar{p}\right)\right] \\
R_A &= R_V + \frac{1}{3}\mu^2\int\int \frac{dxd\bar{x}}{(1-x)(1-\bar{x})}\left[z^2 - 6(1-z) + \frac{3}{2}\mu^2\lambda_+\right] \\
R_F &= \frac{2}{3}\int\int \frac{dxd\bar{x}}{(1-x)(1-\bar{x})}\left[xp - \bar{x}\bar{p} - \frac{1}{2}\mu^2 z\left(\frac{p}{1-x} - \frac{\bar{p}}{1-\bar{x}}\right)\right]
\end{aligned}
$$

Here, x and $\bar{x}$ denote the quark and antiquark energies in units of the beam energy, $z = 2-x-\bar{x}$ is the scaled gluon energy, $\chi = 1/\bar{\chi} = (1-x)/(1-\bar{x})$ and $\lambda_\pm = \chi + \bar{\chi} \pm 2$. p and $\bar{p}$ are the momenta of the quark and antiquark relative to the reference axis and $p_\perp$ is the component of the antiquark momentum perpendicular to this direction:

$$
p = \sqrt{x^2-\mu^2} \quad , \quad \bar{p} = \sqrt{\bar{x}^2-\mu^2}\cos\theta_{q\bar{q}} \quad , \quad p_\perp = \sqrt{\bar{x}^2-\mu^2}\sin\theta_{q\bar{q}}
$$

with

$$
\cos\theta_{q\bar{q}} = \frac{2(1-x-\bar{x}) + x\bar{x} + \mu^2}{\sqrt{x^2-\mu^2}\,\sqrt{\bar{x}^2-\mu^2}}
$$

Using the more convenient variables z and $w = x - \bar{x}$ the phase space integration has to be performed over $|w| < z\left[1-\mu^2/(1-z)\right]^{1/2}$ and $2\lambda < z < 1-\mu^2$.

The first contribution in Eq. (13) accounts for the QCD corrections of the genuine vector–axial vector interference term while the second part is due to the corrections to the total cross section.

However, due to fragmentation effects, cascade decays and detector imperfections, the quark direction cannot always be determined very well. In this realistic situation, the thrust axis lends itself to the reference axis of the forward–backward asymmetry. While the thrust axis coincides with the quark/antiquark line in quark–pair production to lowest order, this connection is destroyed by hard–gluon emission. In fact, with a small probability the gluon carries even more energy than the quarks/antiquarks in three–jet events.

The experimental procedure of choosing the thrust axis and its orientation introduced in Ref. [14], coincides with the definition of the reference axis in this theoretical analysis. The orientation of the thrust axis $\vec{T}$ is defined in such a way that $\vec{T}\cdot\vec{p}$ is positive where $\vec{p}$ is the 3-momentum of the b quark (Fig. 1).

In the three different regions of the Dalitz plot we have for the momenta $p, \bar{p}$ and $p_\perp$:

- $x > \bar{x}, z$ [reference axis parallel to quark line]

$$
p = \sqrt{x^2-\mu^2} \quad , \quad \bar{p} = \sqrt{\bar{x}^2-\mu^2}\cos\theta_{q\bar{q}} \quad , \quad p_\perp = \sqrt{\bar{x}^2-\mu^2}\sin\theta_{q\bar{q}}
$$

- $\bar{x} > x, z$ [reference axis anti–parallel to antiquark line]

$$
\bar{p} = \sqrt{\bar{x}^2-\mu^2} \quad , \quad p = \sqrt{x^2-\mu^2}\cos\theta_{g\bar{q}} \quad , \quad p_\perp = 0
$$

• $z > x, \bar{x}$ [reference axis anti–parallel to gluon line]

$$p = \sqrt{x^2 - \mu^2} \cos\theta_{qg} \quad , \quad \bar{p} = \sqrt{\bar{x}^2 - \mu^2} \cos\theta_{\bar{q}g} \quad , \quad p_\perp = \sqrt{\bar{x}^2 - \mu^2} \sin\theta_{\bar{q}g}$$

with

$$\cos\theta_{qg} = \frac{2(1-x-z)+xz}{z\sqrt{x^2-\mu^2}} \quad \text{and} \quad \cos\theta_{\bar{q}g} = \frac{2(1-\bar{x}-z)+\bar{x}z}{z\sqrt{\bar{x}^2-\mu^2}}$$

In addition, R_F has to be multiplied by (-1) when the reference axis coincides with the antiquark or gluon axis [change from $\cos\theta$ to $-\cos\theta$]; in R_V, the term $\chi p_\perp^2$ has to be replaced by $\lambda_+ p_\perp^2$ when the gluon axis is chosen as the reference axis. Note that according to our prescription for the orientation of the thrust axis, the first two cases give identical results. Events in which the gluon 3-momentum coincides with the thrust axis, dilute the forward–backward asymmetry since $\int \sigma_F$ integrated over the proper part of the Dalitz plot must vanish for this class of events.

The sum of the virtual soft corrections and the hard gluon bremsstrahlung, integrated over the entire allowed range, will change the tree–level forward– backward asymmetry of single quark–jets to

$$A_{FB}^Q = A_{FB/0}^Q \left[1 - \frac{\alpha_s}{\pi} C_F^Q\right] \tag{11}$$

The coefficients C_F^Q [which are free of any singularities and independent of the gluon energy cut separating the soft from the hard bremsstrahlung] are shown in Table 1 for the production of quarks with nearly zero masses, c-quarks and b-quarks. Two reference directions have been chosen: the quark axis and the thrust axis. The numbers to the left of the bars correspond to the pole masses of the quarks, $m_c = 1.5$ GeV and $m_b = 4.5$ GeV; the numbers to the right to the choice [15] $m_c = 0.7$ GeV and $m_b = 3$ GeV for the c, b mass parameters, the mass values in the $\overline{MS}$ scheme at the scale M_Z. The difference between the two predictions reflects the uncertainties due to the corrections of next–to–leading order which have not been included in this analysis. These uncertainties in the prediction of the FB asymmetries, of order 10^{-4}, are *much* smaller than the anticipated ultimate error of the LEP measurements.

One first notices that the coefficient C_F^Q for a given quark species is smaller when the reference axis is defined as the thrust axis. However, while the difference is about $\sim 10\%$ for light quarks it decreases to $\sim 5\%$ in the case of b quarks with $m_b = 4.5$ GeV. The numerical results in the case where the quark axis is chosen as the reference axis, agree with those given in Refs. [5–7], and the result in the massless case for the thrust axis being the reference axis agrees with Ref. [10] if the pole masses are chosen.

The prediction $(-\alpha_s/\pi)$ for the QCD correction of the FB asymmetry in the massless case has been noticed first in Ref. [5]. The simple coefficient (-1) is a consequence of the fact that the QCD corrections to σ_F vanish so that the FB asymmetry is only affected by the well–known QCD correction $1 + \alpha_s/\pi$ to the total cross section in the denominator.

Note also that in the case where the quark axis is chosen as a reference, the coefficient C_F^Q is $\simeq 20\%$ smaller for b–quarks with $m_b = 4.5$ GeV than for massless quarks [as anticipated

Ref. Axis	$C_F^{m=0}$	C_F^c	C_F^b
Quark	1.00	0.93 | 0.96	0.80 | 0.86
Thrust	0.89	0.86 | 0.88	0.77 | 0.81

Table 1: *Coefficients C_F^Q of $(-\alpha_s/\pi)$ in the single quark–jet forward-backward asymmetry with the reference axis being the quark axis, and for the thrust axis; left: pole quark masses, right: $\overline{MS}$ masses at the scale M_Z.*

from Galilei's law of inertia]. Therefore, although $\mu_b = 2m_b/M_Z \sim 0.1$, quark mass effects are vey important for the QCD corrections. For this choice of the reference axis, the coefficient C_F^Q has first been calculated numerically in Ref. [5] for arbitrary quark masses, later it has been derived analytically [8]. A very good approximation can be obtained by making an expansion[1] in the mass parameter μ [8, 12],

$$C_F^Q \approx 1 - \frac{8}{3}\mu \; + \; \cdots \tag{12}$$

In turn, if the thrust axis is chosen as the reference direction, the mass effects are slightly less pronounced. Note also that it is very difficult to obtain analytical results for C_F^Q in this case due to complicated phase space integrals.

4. Two–jet forward–backward asymmetry

The two–jet asymmetry is defined for all events in which the invariant mass of the jets is less than a fraction $\sqrt{y}$ of the total energy, i.e.

$$\begin{array}{lcl} (p+\bar{p})^2/s < y & \longrightarrow & z > 1 - y \\ (p+p_g)^2/s < y & \longrightarrow & \bar{x} > 1 + \frac{1}{4}\mu^2 - y \\ (\bar{p}+p_g)^2/s < y & \longrightarrow & x > 1 + \frac{1}{4}\mu^2 - y \end{array}$$

The two–jet forward–backward asymmetry may then be written again as

$$A_{FB}^Q|_{2\text{jets}} = A_{FB/0}^Q \left[1 - \frac{\alpha_s}{\pi} C_F^Q \right] \tag{13}$$

[1] Due to a small error in the analytic expressions of the integrated parton cross section of Ref. [11], the approximate formula for the forward–backward asymmetry obtained in Ref. [7] was slightly incorrect. The result was so close to the correct prediction that numerical cross–checks failed to reveal this tiny discrepancy. The effect on the asymmetry was of order 10^{-4} for b quarks and thus about two orders of magnitude below the present experimental error, for c quarks smaller still by another order of magnitude.

with the coefficients C_F^Q given in the Table 2 and 3 for a sample of y values[2]; the reference axis is chosen to be the quark axis in Table 2 and the thrust axis in Table 3. The quark masses have been defined as in Table 1. The entries to the left of the bars again correspond to the pole quark masses, those to the right to the $\overline{MS}$ mass definitions at the scale M_Z.

$y_{\rm cut}$	$C_F^{m=0}$	C_F^c	C_F^b
0.01	0.12	0.06 | 0.09	0.02 | 0.03
0.02	0.21	0.14 | 0.17	0.05 | 0.08
0.04	0.35	0.28 | 0.31	0.14 | 0.20
0.08	0.55	0.48 | 0.52	0.34 | 0.40
0.16	0.81	0.74 | 0.77	0.60 | 0.66
2/3	1.00	0.93 | 0.96	0.80 | 0.86

Table 2: *Coefficient C_F^Q of $(-\alpha_s/\pi)$ in A_{FB}^Q as a function of the invariant mass of two jets; the reference axis is taken to be the quark axis; left: pole masses, right: $\overline{MS}$ masses at the scales M_Z.*

$y_{\rm cut}$	$C_F^{m=0}$	C_F^c	C_F^b
0.01	0.07	0.04 | 0.06	0.02 | 0.03
0.02	0.13	0.10 | 0.11	0.04 | 0.06
0.04	0.23	0.19 | 0.21	0.11 | 0.15
0.08	0.39	0.35 | 0.37	0.26 | 0.30
0.16	0.64	0.60 | 0.62	0.51 | 0.55
2/3	0.89	0.86 | 0.88	0.77 | 0.81

Table 3: *Coefficient C_F^Q of $(-\alpha_s/\pi)$ in A_{FB}^Q as a function of the invariant mass of two jets; the reference axis is taken to be the thrust axis; left: pole masses, right: $\overline{MS}$ masses at the scale M_Z.*

The following conclusions can be drawn from the tables. First, as anticipated, the two–jet coefficient C_F^Q decreases with decreasing y cut, and they become very small for small invariant masses. Two–jet asymmetries are therefore less affected by the QCD corrections and they can be used to reduce the errors coming from the relatively poor knowledge of α_s and from

[2]The entries in Tables 2/3 supersede those given in Ref. [7].

higher–order effects. Furthermore, the coefficient C_F^Q of the two–jet asymmetry follows the same pattern as the asymmetry for single quark jets and for the thrust axis in Table 1: The QCD corrections are smaller in the case where the thrust axis is chosen as the reference axis, than in the case where the reference axis is the quark axis; the corrections decrease also with increasing mass. Finally, for the maximum value of the y cut, $y = 2/3$, there are by definition no three-jet events left over and the two–jet asymmetry approaches the single–jet forward–backward asymmetry.

5. Summary

We have presented an overview of the one–loop QCD corrections to forward–backward asymmetries for heavy–quark production in Z decays. A complete set of results has been given which improve the theoretical predictions for these asymmetries, the measurement of which provides some of the most precise determinations of the electroweak mixing angle $\sin^2\theta_W$ in Z decays at LEP and SLC. We have focussed on the consequences of quark–mass corrections which we have shown to be rather important. The difference between the QCD corrections to the forward–backward asymmetries for light quarks and for b-quarks turns out to be as large as 20%. With the high–accuracy of the measurements performed at LEP, these effects must be taken into account properly.

We have discussed these radiative corrections for the two cases where the reference directions are taken either to be the quark axis or the thrust axis. In the second case the QCD corrections are slighty smaller. We have also considered the consequences of constraints on the jets by demanding their invariant mass to be less than a fraction $\sqrt{y}$ of the total energy. These two-jet asymmetries are affected less by QCD corrections than the single–jet asymmetries, and they could therefore be exploited to minimize the errors due to strong interactions in experimental high–statistics analyses.

Acknowledgement: We would like to thank R. Tenchini for helpful comments. PMZ thanks D. Yu. Bardin and J.B. Stav for clarifying discussions on the expansion of the single–jet FB asymmetries in the quark mass, and M. Bilenky and A. Santamaria for a correspondence.

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Figure Caption

Fig. 1 Definition of the oriented thurst axis $\vec{T}$ in 3-jet events.

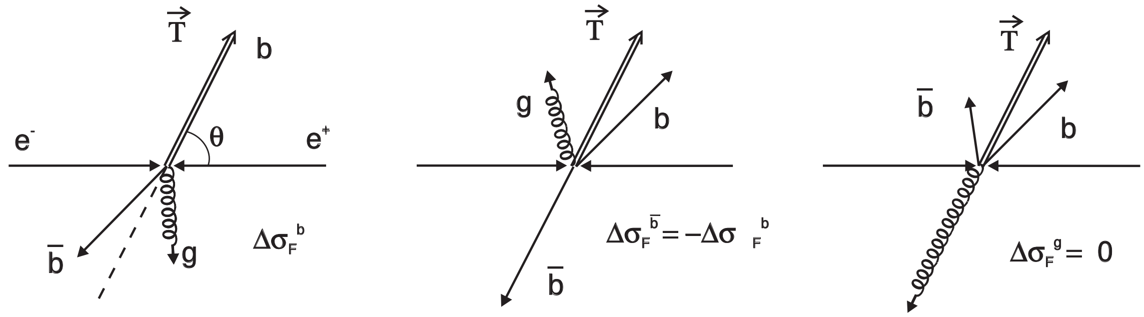

Fig. 1